Modelling Multiple Perspectives by Standpoint-Enhanced Description Logics

Extended Abstract

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1. Introduction

Ontologies and knowledge bases reflect not only domain specific knowledge, but also the individual points of view of their creators along with other contextual aspects. They may also differ in modelling design decisions, such as the choice of conceptual granularity or specific ways of axiomatising information. This semantic heterogeneity is bound to pose significant challenges whenever the interoperability of independently developed knowledge specifications is required.

Example 1. *Consider the integration of two ontologies of forestry* LC *and* LU*, adopting a* land cover *and a* land use *perspective respectively.* LC *characterises a* forest *as a "forest ecosystem" with a minimum area (G1) where a* "forest ecosystem" *is an ecosystem with a certain ratio of tree canopy cover (G2). In contrast,* LU *defines a forest as a maximally connected area with "forest use" (G3). Both sources* LC *and* LU *agree that forests subsume broadleaf, needleleaf and tropical forests (G4), and they both adhere to the Basic Formal Ontology (*BFO*, Arp et al. 2015), an upper-level ontology stipulating that "land" and "ecosystem" are disjoint categories* (G5).

(G1) $\square_{\mathsf{LC}}[\mathtt{Forest} \equiv \mathtt{ForestEcosystem} \sqcap \exists \mathtt{hasLand.Area}_{\geq 0.5\mathrm{ha}}]$
(G2) $\square_{\mathsf{LC}}[\mathtt{ForestEcosystem} \equiv \mathtt{Ecosystem} \sqcap \mathtt{TreeCanopy}_{\geq 20\%}]$
(G3) $\square_{\mathsf{LU}}[\mathtt{Forest} \equiv \mathtt{ForestlandUse} \sqcap \mathtt{MCON}] \ \wedge \ \square_{*}[\mathtt{ForestlandUse} \sqsubseteq \mathtt{Land}]$
(G4) $\square_{\mathsf{LC}\cup\mathsf{LU}}[(\mathtt{BroadleafForest} \sqcup \mathtt{NeedleleafForest} \sqcup \mathtt{TropicalForest}) \sqsubseteq \mathtt{Forest})]$
(G5) $(\mathsf{LC} \cup \mathsf{LU} \preceq \mathsf{BFO}) \wedge \square_{\mathsf{BFO}}[(\mathtt{Land} \sqcap \mathtt{Ecosystem}) \sqsubseteq \bot]$

Ontology merging focuses on combining and merging different sources into a single conflict-free conceptual model, a non-trivial task [2, 3] often involving a certain knowledge loss or weakening in order to avoid incoherence and inconsistency [4, 5]. For instance, in Example 1, forests are defined as ecosystems in LC (G1) and as lands in LU (G3), with *ecosystem* and *land* being disjoint categories (G5). To merge LU and LC, one would typically (Opt-Weak) give up on the disjointness axiom (G5), or (Opt-Dup) duplicate the conflicting predicates [6], here both

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`Forest` and all its subclasses (G4). As an alternative, we propose Standpoint Logic, a simple, yet versatile generic approach to extend existing KR formalisms by the capability to express domain knowledge relative to diverse, possibly conflicting standpoints.

In multimodal *Standpoint logic* [7] propositions of the form $\Box_{\mathsf{s}}\,\phi$ and $\Diamond_{\mathsf{s}}\,\phi$ express information relative to the *standpoint* s and read, respectively: "according to s, it is *unequivocal/conceivable* that ϕ". In the semantics, inspired by the theory of supervaluationism [8], standpoints are represented by sets of *precisifications* (akin to worlds in possible world semantics), such that $\Box_{\mathsf{s}}\,\phi$ and $\Diamond_{\mathsf{s}}\,\phi$ hold if ϕ is true in all/some of the precisifications in s. This makes the single-modal standpoint logic close to the modal logic K45 [9], where the usual Kripke relations can be simplified into sets. Unlike in K45, in standpoint logic this simplification can be carried over to the multi-modal case, and in addition combinations of standpoints are allowed, as in (G4).

In spite of its simple syntax and semantics, the language is remarkably versatile; it allows for specifying knowledge relative (a) to a standpoint, e.g. (G1), (b) to the global standpoint, denoted by $*$, e.g. (G3), and (c) to set-theoretic combinations of standpoints, e.g. (G4). Additional definabe constructs include $\mathcal{I}_{\mathsf{LC}\cup\mathsf{LU}}[\mathtt{Forest} \sqsubseteq \mathtt{Land}]$, meaning that, "according to $\mathsf{LC} \cup \mathsf{LU}$, it is *indeterminate* whether land subsumes forest" (with $\mathcal{I}_{\mathsf{s}}\phi := \Diamond_{\mathsf{s}}\,\neg\phi \wedge \Diamond_{\mathsf{s}}\,\phi$). The *sharper* operator $\preceq$, used to establish hierarchies of standpoints, can be defined via $\mathsf{s}_1 \preceq \mathsf{s}_2 := \Box_{\mathsf{s}_1\setminus\mathsf{s}_2}[\top \sqsubseteq \bot]$. Intuitively, $\mathsf{s}_1 \preceq \mathsf{s}_2$ expresses that standpoint s_1 inherits the propositions of s_2, by virtue of "$\mathsf{s}_1 \subseteq \mathsf{s}_2$" holding for the corresponding sets of precisifications. This is used in the example to "import" background knowledge from the upper-level ontology BFO (G5) and it can be used in the axiom $* \preceq (\mathsf{LC} \cup \mathsf{LU})$ to confine the interest to a given set of standpoints (each precisification must be associated with at least one of those). Natural reasoning tasks over multi-standpoint specifications include gathering unequivocal or undisputed knowledge, determining knowledge that is relative to a standpoint or a set of them, and contrasting the knowledge that can be inferred from different standpoints.

This extended abstract introduces the standpoint-enhanced version of the very expressive description logic $\mathcal{SROIQ}b_s$ (cf. Section 2.2), which is tightly connected to the W3C-standardised ontology language OWL 2 DL. Here, standpoint modalities are allowed only at the axiom level, which – coupled with the semantics of the multi-modal standpoint logic framework – makes it possible to establish a polytime translation from standpoint-enhanced $\mathcal{SROIQ}b_s$ into plain $\mathcal{SROIQ}b_s$ (cf. Section 2.3). By virtue of this result, existing highly optimised OWL reasoners can be used off the shelf to provide reasoning support for ontology languages from the OWL family extended by standpoint modelling.

2. Sentential Standpoint-$\mathcal{SROIQ}b_s$

Sentential Standpoint-$\mathcal{SROIQ}b_s$ ($\mathbb{S}_{[\mathcal{SROIQ}b_s]}$) adds the standpoint modalities to $\mathcal{SROIQ}b_s$ at the sentence (axiom) level. After an introduction to $\mathcal{SROIQ}b_s$ (Section 2.1), we provide the syntax and semantics of $\mathbb{S}_{[\mathcal{SROIQ}b_s]}$ (Section 2.2), we state a small model property and finally we present a satisfiability-preserving polynomial translation from $\mathbb{S}_{[\mathcal{SROIQ}b_s]}$ into plain $\mathcal{SROIQ}b_s$ (Section 2.3). This provides us with tight complexity results and it paves the way towards practical reasoning in "Standpoint-OWL", since it allows us to use highly optimised OWL 2 DL reasoners.

2.1. $\mathcal{SROIQ}b_s$

$\mathcal{SROIQ}b_s$ is a gentle extension of $\mathcal{SROIQ}$ [10] allowing safe Boolean role expressions over simple roles [11]. By focusing on the mildly stronger $\mathcal{SROIQ}b_s$ instead of the more mainstream $\mathcal{SROIQ}$, we can provide a more coherent and economic presentation, without giving up the good computational properties and the availability of optimised algorithms and tools.

In what follows, let $N_{\mathbf{I}}$, $N_{\mathbf{C}}$, and $N_{\mathbf{R}}$ be finite, mutually disjoint sets called *individual names*, *concept names* and *role names*, respectively. $N_{\mathbf{R}}$ is subdivided into *simple role names* $N_{\mathbf{R}}^{\mathrm{s}}$ and *non-simple role names* $N_{\mathbf{R}}^{\mathrm{ns}}$, the latter containing the *universal role* $\mathtt{u}$ and being strictly ordered by $\lessdot$. We briefly mention that unlike in the original definition of $\mathcal{SROIQ}b_s$, we fix simplicity of roles and $\lessdot$ explicitly upfront, simplifying the presentation without restricting expressivity. Then, let $\mathcal{R}^{\mathrm{s}}$ be the set of *simple role expressions* such that $r_1, r_2 ::= \mathtt{s} \mid \mathtt{s}^- \mid r_1 \cup r_2 \mid r_1 \cap r_2 \mid r_1 \setminus r_2$, with $\mathtt{s} \in N_{\mathbf{R}}^{\mathrm{s}}$, and let $\mathcal{R} = \mathcal{R}^{\mathrm{s}} \cup N_{\mathbf{R}}^{\mathrm{ns}}$ be the set of (arbitrary) *role expressions*. The order $\lessdot$ is extended to $\mathcal{R}$ by making all elements of $\mathcal{R}^{\mathrm{s}}$ $\lessdot$-minimal. The syntax and semantics of *concept* and *role expressions* as well as the different types of $\mathcal{SROIQ}b_s$ sentences (called *axioms*) are defined as usual [10, 11]. Finally, any concept expression can be put in NNF, where negation only occurs in front of concept names, nominals, or Self concepts.

2.2. Sentential Standpoint-$\mathcal{SROIQ}b_s$: Syntax and Semantics

Given a set $\mathcal{S}$ of *standpoint symbols* containing the *universal standpoint* $*$, we define the set $\mathcal{E}_{\mathcal{S}}$ of *standpoint expressions* via $\mathsf{e}_1, \mathsf{e}_2 ::= \mathsf{s} \mid \mathsf{e}_1 \cup \mathsf{e}_2 \mid \mathsf{e}_1 \cap \mathsf{e}_2 \mid \mathsf{e}_1 \setminus \mathsf{e}_2$, where $\mathsf{s} \in \mathcal{S}$. Then, the set $\mathbb{S}_{[\mathcal{SROIQ}b_s]}$ of *Sentential Standpoint-$\mathcal{SROIQ}b_s$ sentences* is defined inductively over $\mathcal{E}_{\mathcal{S}}$ and the set of $\mathcal{SROIQ}b_s$ axioms:

- if $\mathbf{Ax}$ is a $\mathcal{SROIQ}b_s$ axiom then $\mathbf{Ax} \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$,
- if $\phi, \psi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$ then $\neg\phi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$, $\phi \wedge \psi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$, and $\phi \vee \psi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$,
- if $\phi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$ and $\mathsf{e} \in \mathcal{E}_{\mathcal{S}}$ then $\Box_{\mathsf{e}}\, \phi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$ and $\Diamond_{\mathsf{e}}\, \phi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$.

In what follows, we assume that formulas are in SSNF, whereby their modal degree is at most 1.

The semantics of sentential Standpoint-$\mathcal{SROIQ}b_s$ is defined by "plugging" the semantics of $\mathcal{SROIQ}b_s$ axioms into a multi-modal semantics where precisifications replace worlds and the usual set of accessibility relations is replaced by the function σ. Thus, a *Standpoint-$\mathcal{SROIQ}b_s$ structure* $\mathfrak{M}$ is a tuple $\langle \Delta, \Pi, \sigma, \gamma \rangle$ where (a) Δ is a non-empty set, the *domain* of $\mathfrak{M}$, (b) Π is the non-empty set of *precisifications*, (c) σ is a function mapping each standpoint symbol to a set of precisifications, with $\sigma(*) = \Pi$, and (d) γ is a function mapping each precisification from Π to an ordinary $\mathcal{SROIQ}b_s$ interpretation $\mathcal{I} = (\Delta, \cdot^{\mathcal{I}})$ composed of the (shared[1]) domain Δ and a function $\cdot^{\mathcal{I}}$ mapping individual names to elements of Δ, concept names to subsets of Δ, and role names to subsets of $\Delta \times \Delta$.

Theorem 1. *A sentential Standpoint-$\mathcal{SROIQ}b_s$ formula ϕ is satisfiable iff it has a model with at most $|\phi|$ precisifications.*

Theorem 1 can be shown following a similar method to the propositional case [7] because the use of modalities exclusively at the axiom level prevents the interaction between the modal operators and implicit free variables. Proofs can be found at https://arxiv.org/abs/2206.06793.

[1] We adopt the *constant domain assumption*, with which expanding and varying domains can be emulated [12].

2.3. Translation into Plain $\mathcal{SROIQ}b_s$

To conclude this short paper, we provide a polynomial translation mapping any $\mathbb{S}_{[\mathcal{SROIQ}b_s]}$ sentence ϕ (w.l.o.g. in NNF) to an equisatisfiable set of $\mathcal{SROIQ}b_s$ axioms. With this, we establish the decidability and complexity of $\mathbb{S}_{[\mathcal{SROIQ}b_s]}$ and we expose a strategy to support standpoint extensions of languages of the OWL family with off-the-shelf reasoners.

Let $\Pi_{|\phi|}$ be a set of size $|\phi|$ (Theorem 1) and our translation's vocabulary $\mathbb{V}_{[\mathcal{SROIQ}b_s]}(\phi)$ consist of all individual names inside ϕ, plus, for each $\pi \in \Pi_{|\phi|}$, the following symbols: a concept name $\mathtt{A}^\pi$ for each $\mathtt{A} \in N_\mathbf{C}$; a simple role name $\mathtt{s}^\pi$ for each $\mathtt{s} \in N_\mathbf{R}^\mathrm{s}$; non-simple role names $\mathtt{r}^\pi$ and $\underline{\mathtt{r}}^\pi$ for each $\mathtt{r} \in N_\mathbf{R}^\mathrm{ns} \setminus \{\mathtt{u}\}$; a simple role name $\mathtt{s}_\rho^\pi$ for each unnegated RIA ρ inside ϕ; a fresh constant name a_ρ^π for each negated RIA ρ inside ϕ; and a concept name $\mathtt{M}_\pi^\mathtt{s}$ for each $\mathtt{s} \in \mathcal{S}$. The non-simple role names inherit their ordering $\lessdot$ from $N_\mathbf{R}^\mathrm{ns}$ and we let $\underline{\mathtt{r}}^\pi \lessdot \mathtt{r}^\pi$ for each $\mathtt{r} \in N_\mathbf{R}^\mathrm{ns} \setminus \{\mathtt{u}\}$.

The translation $\mathrm{Tr}(\phi)$ of ϕ is then a set of $\mathcal{SROIQ}$ axioms defined as follows: First, $\mathrm{Tr}(\phi)$ contains the RIA $\underline{\mathtt{r}}^\pi \sqsubseteq \mathtt{r}^\pi$ for every $\mathtt{r} \in N_\mathbf{R}^\mathrm{ns} \setminus \{\mathtt{u}\}$ and each $\pi \in \Pi_{|\phi|}$. Second, for every unnegated RIA ρ inside ϕ and each $\pi \in \Pi_{|\phi|}$, $\mathrm{Tr}(\phi)$ contains the RIA $BG_\pi(\rho)$, with BG_π defined by

$$r_1 \circ \ldots \circ r_n \sqsubseteq \mathtt{r} \mapsto \mathtt{s}_\rho^\pi \circ r_1^\pi \circ \ldots \circ r_n^\pi \sqsubseteq \underline{\mathtt{r}}^\pi \qquad r_1 \circ \ldots \circ r_n \circ \mathtt{r} \sqsubseteq \mathtt{r} \mapsto \mathtt{s}_\rho^\pi \circ r_1^\pi \circ \ldots \circ r_n^\pi \circ \underline{\mathtt{r}}^\pi \sqsubseteq \underline{\mathtt{r}}^\pi$$
$$\mathtt{r} \circ r_1 \circ \ldots \circ r_n \sqsubseteq \mathtt{r} \mapsto \underline{\mathtt{r}}^\pi \circ r_1^\pi \circ \ldots \circ r_n^\pi \circ \mathtt{s}_\rho^\pi \sqsubseteq \underline{\mathtt{r}}^\pi \qquad \mathtt{r} \circ \mathtt{r} \sqsubseteq \mathtt{r} \mapsto \mathtt{s}_\rho^\pi \circ \underline{\mathtt{r}}^\pi \circ \mathtt{r}^\pi \sqsubseteq \mathtt{r}^\pi,$$

whereby the role expression r^π is obtained from r by substituting every role name $\mathtt{s}$ with $\mathtt{s}^\pi$ (except $\mathtt{u}$ which remains unaltered). Third and last, $\mathrm{Tr}(\phi)$ contains the GCI

$$\top \sqsubseteq \textstyle\bigsqcap_{\pi \in \Pi_{|\phi|}} \mathrm{tr}(\pi, \phi) \sqcap \bigsqcap_{\pi \in \Pi_{|\phi|}} \forall \mathtt{u}.\mathtt{M}_\pi^*$$

where, by inductive definition,

$$\mathrm{tr}(\pi, \mathbf{Ax}) = \mathrm{tr}^+(\pi, \mathbf{Ax}) \qquad \mathrm{tr}(\pi, \psi_1 \vee \psi_2) = \mathrm{tr}(\pi, \psi_1) \sqcup \mathrm{tr}(\pi, \psi_2)$$
$$\mathrm{tr}(\pi, \neg\mathbf{Ax}) = \mathrm{tr}^-(\pi, \mathbf{Ax}) \qquad \mathrm{tr}(\pi', \Box_\mathsf{e}\, \psi) = \textstyle\bigsqcap_{\pi \in \Pi_{|\phi|}} (\neg \mathrm{tr}_\mathcal{E}(\pi, \mathsf{e}) \sqcup \mathrm{tr}(\pi, \psi))$$
$$\mathrm{tr}(\pi, \psi_1 \wedge \psi_2) = \mathrm{tr}(\pi, \psi_1) \sqcap \mathrm{tr}(\pi, \psi_2) \qquad \mathrm{tr}(\pi', \Diamond_\mathsf{e}\, \psi) = \textstyle\bigsqcup_{\pi \in \Pi_{|\phi|}} (\mathrm{tr}_\mathcal{E}(\pi, \mathsf{e}) \sqcap \mathrm{tr}(\pi, \psi))$$

The translation of unnegated and negated $\mathcal{SROIQ}$ axioms

$$\mathrm{tr}^+(\pi, \rho) = \forall \mathtt{u}.\exists \mathtt{s}_\rho^\pi.\mathit{Self} \qquad \mathrm{tr}^-(\pi, \rho) = \exists \mathtt{u}.\big((\forall \mathtt{r}^\pi.\neg\{a_\rho^\pi\}) \sqcap (\exists \underline{r}_1^\pi \ldots \exists \underline{r}_m^\pi.\{a_\rho^\pi\})\big)$$
$$\mathrm{tr}^+(\pi, C \sqsubseteq D) = \forall \mathtt{u}.(\neg C \sqcup D)^\pi \qquad \mathrm{tr}^-(\pi, C \sqsubseteq D) = \exists \mathtt{u}.(C \sqcap \neg D)^\pi$$
$$\mathrm{tr}^+(\pi, C(a)) = \exists \mathtt{u}.\big(\{a\} \sqcap C^\pi\big) \qquad \mathrm{tr}^-(\pi, C(a)) = \exists \mathtt{u}.\big(\{a\} \sqcap (\neg C)^\pi\big)$$
$$\mathrm{tr}^+(\pi, r(a,b)) = \exists \mathtt{u}.\big(\{a\} \sqcap \exists \underline{r}^\pi.\{b\}\big) \qquad \mathrm{tr}^-(\pi, r(a,b)) = \exists \mathtt{u}.\big(\{a\} \sqcap \forall r^\pi.\neg\{b\}\big)$$
$$\mathrm{tr}^+(\pi, a \doteq b) = \exists \mathtt{u}.\big(\{a\} \sqcap \{b\}\big) \qquad \mathrm{tr}^-(\pi, a \doteq b) = \exists \mathtt{u}.\big(\{a\} \sqcap \neg\{b\}\big)$$

Therein, for any role expression r, we let $\underline{r}$ denote $\underline{\mathtt{r}}$ if $r = \mathtt{r}$ is a non-simple role name, and otherwise $\underline{r} = r$. Moreover, C^π denotes the concept expression that is obtained from C by first transforming it into negation normal form, second replacing concept names $\mathtt{A}$ with $\mathtt{A}^\pi$ and role expressions $\mathtt{r}$ by $\mathtt{r}^\pi$, and third replacing every $\exists\mathtt{r}$ for non-simple $\mathtt{r}$ with $\exists\underline{\mathtt{r}}$. Finally, $\mathrm{tr}_\mathcal{E}$ implements the semantics of standpoint expressions: Each expression $\mathsf{e} \in \mathcal{E}_\mathcal{S}$ is transformed into a concept expression $\mathrm{tr}_\mathcal{E}(\pi, \mathsf{e})$ over the vocabulary $\big\{\mathtt{M}_\pi^\mathtt{s} \,|\, s \in \mathcal{S}, \pi \in \Pi_{|\phi|}\big\}$ thus:

$$\mathrm{tr}_\mathcal{E}(\pi, \mathsf{s}) = \forall \mathtt{u}.\mathtt{M}_\pi^\mathtt{s} \qquad \mathrm{tr}_\mathcal{E}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) = \mathrm{tr}_\mathcal{E}(\pi, \mathsf{e}_1) \sqcup \mathrm{tr}_\mathcal{E}(\pi, \mathsf{e}_2)$$
$$\mathrm{tr}_\mathcal{E}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) = \mathrm{tr}_\mathcal{E}(\pi, \mathsf{e}_1) \sqcap \mathrm{tr}_\mathcal{E}(\pi, \mathsf{e}_2) \qquad \mathrm{tr}_\mathcal{E}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) = \mathrm{tr}_\mathcal{E}(\pi, \mathsf{e}_1) \sqcap \neg\mathrm{tr}_\mathcal{E}(\pi, \mathsf{e}_2)$$

With all definitions in place, we obtain the desired result, which concludes the paper.

Theorem 2. *Given $\phi \in \mathbb{S}_{[\mathcal{SROIQ}b_s]}$, the set $\mathrm{Tr}(\phi)$ is a valid $\mathcal{SROIQ}b_s$ knowledge base, is equisatisfiable with ϕ, is of polynomial size wrt. ϕ, and can be computed in polynomial time.*

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